

Exercise 34Find $f'(a)$.

$$f(x) = x^{-2}$$

SolutionDetermine the derivative of $f(x)$.

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{(x+h)^{-2} - x^{-2}}{h} \\ &= \lim_{h \rightarrow 0} \frac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h} \\ &= \lim_{h \rightarrow 0} \frac{\frac{x^2}{x^2(x+h)^2} - \frac{(x+h)^2}{x^2(x+h)^2}}{h} \\ &= \lim_{h \rightarrow 0} \frac{\frac{x^2 - (x+h)^2}{x^2(x+h)^2}}{h} \\ &= \lim_{h \rightarrow 0} \frac{x^2 - (x+h)^2}{hx^2(x+h)^2} \\ &= \lim_{h \rightarrow 0} \frac{x^2 - (x^2 + 2xh + h^2)}{hx^2(x+h)^2} \\ &= \lim_{h \rightarrow 0} \frac{-2xh - h^2}{hx^2(x+h)^2} \\ &= \lim_{h \rightarrow 0} \frac{-2x - h}{x^2(x+h)^2} \\ &= \frac{-2x}{x^2(x)^2} \\ &= -\frac{2}{x^3} \end{aligned}$$

Plug in $x = a$ to this formula to get $f'(a)$.

$$f'(a) = -\frac{2}{a^3}$$